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ON ONE DIMENSIONAL GEOSTROPHIC
ADJUSTMENT WITH FINITE DIFFERENCING

by

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Winninghoff [6], in his doctoral thesis investigated the effect on the process of geostrophic adjustment [2] in one dimension of several different spatial discretization schemes. His results were later incorporated by Arakawa and Lamb [1]. The model he chose, which had been previously investigated by Rossby [4], Cahn [3] and others, was the linearized one-dimensional shallow water equations with no mean flow, in an infinite region:

$$\frac{\partial u}{\partial t} - fv + g \frac{\partial h}{\partial x} = 0 \quad (1)$$

$$\frac{\partial v}{\partial t} + fu = 0 \quad (2)$$

$$\frac{\partial h}{\partial t} + H \frac{\partial u}{\partial x} = 0 , \quad (3)$$

where u is the perturbation velocity in the x direction, v is the perturbation velocity normal to the x direction, H and h are the mean and perturbed heights of the free surface, respectively, and $g > 0$ and $f > 0$ are gravitational and Coriolis parameters, respectively. The purpose of this note is to point out certain errors in Winninghoff's solutions, present the correct solutions, and reexamine his conclusions in light of the correct solutions.

Winninghoff followed earlier studies in solving (1)-(3) by eliminating between the equations to arrive at:

$$\frac{\partial^2 u}{\partial t^2} + f^2 u - gH \frac{\partial^2 u}{\partial x^2} = 0 , \quad (4)$$

then solving (4) by a Fourier Transform approach. After solving (4), solutions for h and v are obtained by substitution into (2) and (3),

although closed-form solutions are not presented in some of the papers. Note the dispersive character of (4) is clearly seen by assuming a wave solution:

$$u(x,t) = A e^{i(kx-vt)}$$

which implies:

$$v^2 = f^2 + gHk^2 = f^2 (1 + \lambda^2 k^2), \quad \lambda = \sqrt{gH/f} . \quad (5)$$

In [5], we presented a method where (1)-(3) were transformed directly, then the transformed problem was solved as a system of coupled ordinary differential equations.¹ If we denote transforms by an overhead tilde, e.g.

$$\tilde{u}(k,t) = \int_{-\infty}^{\infty} u(x,t) e^{-ikx} dx ,$$

then (1)-(3) reduce to

$$\begin{aligned} \frac{d\tilde{u}}{dt} &= f\tilde{v} - ikg \tilde{h} \\ \frac{d\tilde{v}}{dt} &= -f\tilde{u} \\ \frac{d\tilde{h}}{dt} &= -ikH \tilde{u} . \end{aligned} \quad (6)$$

Now, if we denote initial values by a subscript, e.g.

$$\tilde{u}_0(k) = \int_{-\infty}^{\infty} \tilde{u}(x,0) e^{-ikx} dx = \tilde{u}(k,0) ,$$

and use Winninghoff's initial conditions:

$$\begin{aligned} u(x,0) &= 0 \\ h(x,0) &= 0 \\ v(x,0) &= \begin{cases} v_0, & -a < x < a \\ 0, & \text{otherwise} , \end{cases} \end{aligned} \quad (7)$$

then using equation (12) of [5], the transformed solution is:

$$\begin{aligned}
\tilde{u}(k,t) &= \frac{2fv_0}{v} \sin(vt) \frac{\sin(ak)}{k} \\
\tilde{v}(k,t) &= 2V_0 \left\{ \frac{k^2 gH}{v^2} + \frac{f^2}{v^2} \cos(vt) \right\} \frac{\sin(ak)}{k} \\
\tilde{h}(k,t) &= - \frac{2ikHfv_0}{v^2} \{1 - \cos(vt)\} \frac{\sin(ak)}{k} .
\end{aligned} \tag{8}$$

Winninghoff, using elimination, solved only for $u(x,t)$,

$$\begin{aligned}
u(x,t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{u}(k,t) e^{ikx} dk \\
&= \frac{2faV_0}{\pi} \int_0^{\infty} \frac{\sin(ak)}{ak} \frac{\sin(vt)}{v} \cos kx dk ,
\end{aligned} \tag{9}$$

then solved for $h(x,t)$ using the continuity equation. [If this is done to (9), or if the third equation in (8) is inverted directly, we find

$$h(x,t) = \frac{2Hfv_0}{\pi} \int_0^{\infty} \frac{\{1 - \cos(vt)\}}{v^2} \sin(ak) \sin(kx) dk . \tag{10}]$$

The error in his solution occurs in converting from the differential case to the discretized case, using any one of the difference schemes he denotes as schemes A-E. (Figure 1) He performs this conversion by simply replacing v in (9) by its equivalent finite difference form, evaluating the integral numerically, then finding $h(x,t)$ from the continuity equation. We now show this approach neglects the effects of spatial averaging of $u(x,t)$ and $v(x,t)$ that occurs in Schemes C and D. For example, in Scheme C, the finite difference form of (1)-(3) is:

$$\left. \begin{aligned}
\frac{\partial u}{\partial t} &= f \frac{[v(x+d/2,t) + v(x-d/2,t)]}{2} - g \frac{[h(x+d/2,t) - h(x-d/2,t)]}{d} \\
\frac{\partial v}{\partial t} &= -f \frac{[u(x+d/2,t) + u(x-d/2,t)]}{2} \\
\frac{\partial h}{\partial t} &= -H \frac{[u(x+d/2,t) - u(x-d/2,t)]}{d} .
\end{aligned} \right\} \tag{11}$$

If we transform this system, distinguishing from the differential system by using a subscript c on the variables, we have:

$$\begin{aligned}\frac{d\tilde{u}_c}{dt} &= f \cos(kd/2) \tilde{v}_c - ig \frac{\sin(kd/2)}{(d/2)} \tilde{h}_c \\ \frac{d\tilde{v}_c}{dt} &= -f \cos(kd/2) \tilde{u}_c \\ \frac{\partial \tilde{h}_c}{\partial t} &= -iH \frac{\sin(kd/2)}{(d/2)} \tilde{u}_c.\end{aligned}\tag{12}$$

This system is identical to (6), except that f is replaced by $f \cos(kd/2)$ and k is replaced by $\sin(kd/2)/(d/2)$. Thus the solutions for $u_c(x,t)$ and $h_c(x,t)$, corresponding to Arakawa's initial condition, can be obtained by making a similar replacement in (12) of [5], to yield:

$$u_c(x,t) = \frac{2faV_0}{\pi} \int_0^\infty \frac{\sin(ak)}{ak} \frac{\sin(v_c t)}{v_c} \cos(kx) \cos(kd/2) dk, \tag{13}$$

$$h_c(t) = \frac{2HfaV_0}{\pi} \int_0^\infty \frac{\{1 - \cos(v_c t)\}}{v_c^2} \frac{\sin(ak)}{ak} \sin(kx) \frac{\sin(kd/2) \cos(kd/2)}{(d/2)} dk, \tag{14}$$

where

$$v_c^2 = f^2(1 + 4(\lambda/d)^2 \sin^2(kd/2)). \tag{15}$$

Observe that (13) is not obtainable from (9) simply by replacing v by v_c . This is because of the $\cos(kd/2)$ term that arises from the spatial averaging of v and u that occurs in the first and second equations of (11), respectively. Repeating these calculations for the other schemes will show that $u(x,t)$ in Scheme D also requires a $\cos(kd/2)$ term, and that $h(x,t)$ is given, in general, by

$$h(x,t) = \frac{2HfaV_0}{\pi} \int_0^\infty D(k) \frac{\{1 - \cos vt\}}{v^2} \frac{\sin(ak)}{ak} \sin(kx) dk \tag{16}$$

where

$$D(k) = \left\{ \begin{array}{ll} k & , \text{ Differential} \\ \frac{\sin(kd)}{d} & , \text{ Scheme A} \\ \frac{\sin(kd/2)}{(d/2)} & , \text{ Scheme B} \\ \frac{\cos(kd/2)\sin(kd/2)}{(d/2)} & , \text{ Scheme C} \\ \frac{\cos^2(kd/2)\sin(kd/2)}{(d/2)} & , \text{ Scheme D} \\ \frac{\sin(kd/\sqrt{2})}{(d/\sqrt{2})} & , \text{ Scheme E} \end{array} \right. \quad (17)$$

and

$$v/f = \left\{ \begin{array}{ll} [1 + \lambda^2 k^2]^{1/2} & , \text{ Differential} \\ [1 + (\lambda/d)^2 \sin^2(kd)]^{1/2} & , \text{ Scheme A} \\ [1 + 4(\lambda/d)^2 \sin^2(kd/2)]^{1/2} & , \text{ Scheme B} \\ [\cos^2(kd/2) + 4(\lambda/d)^2 \sin^2(kd/2)]^{1/2} & , \text{ Scheme C} \\ [\cos^2(kd/2) + (\lambda/d)^2 \sin^2(kd)]^{1/2} & , \text{ Scheme D} \\ [1 + 2(\lambda/d)^2 \sin^2(kd/\sqrt{2})]^{1/2} & , \text{ Scheme E} \end{array} \right. \quad (18)$$

In Figure 2, the integrals given by (16) are used to compute (h/H) at $t = 80$ hours in the same manner as done by Winninghoff, i.e., using Simpson's rule with 600 subintervals in the interval $(0, \pi/d)$. The relevant parameter values used were

$$f = 10^{-4} \text{ sec}^{-1}$$

$$g = 10 \text{ m sec}^{-2}$$

$$H = 10^3 \text{ m}$$

$$a = d = \lambda/2 .$$

These graphs too should be compared to Figure 3, which shows the equivalent evaluation of (10), with only $v(k)$ replaced by its equivalent finite difference value. (Figure 3 appears identical to Figure 3.6 in Winninghoff.) Note especially the effect on the solution to Scheme D. We conclude that Winninghoff obtained such ill-behaved solutions in this case because he neglected the $\cos(kd/2)$ term in the numerator of $u(x,t)$ in these cases, and this term helps to control the singular behavior of $[v(k)]^{-1}$ as $k \rightarrow \pi$ in Scheme D.

Winninghoff eliminated Scheme D from further study due to the extremely noisy behavior shown by Figure 3. Since his calculation was erroneous, it can not be discarded for this reason, however, as he showed, it is still seriously deficient in terms of phase speed errors, and should probably be discarded for this reason.

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FOOTNOTE

1. A sgn term was erroneously omitted from certain expressions in [5]. Thus, in [5], equation (13) should read,

$$h_S(x) = h(x,0) - \frac{H}{2\lambda^2 f} \int_{-\infty}^{\infty} \text{sgn}(x-s) e^{-|x-s|/\lambda} \left\{ \frac{g}{f} \frac{\partial h}{\partial x}(s,0) - v(s,0) \right\} ds ,$$

the equation immediately above (28) should read

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{i\sigma}{1 + \lambda^2 \sigma^2} e^{ikx} dk = - \frac{\Delta x e^{-\beta|x|\lambda} \text{sgn}(x)}{\lambda [\lambda^2 + (\Delta x)^2]^{1/2}} \sum_{n=-\infty}^{\infty} \delta(x - (2n-1)\Delta x) ,$$

equation (30) should read

$$h_S(x,0) = h(x,0) + \frac{H}{2\lambda f [\lambda^2 + (\Delta x)^2]^{1/2}} \\ \times \sum_{n=-\infty}^{\infty} \{ \text{sgn}((2n-1)\Delta x) e^{-\beta|(2n-1)\Delta x|/\lambda} \hat{d}(x - (2n-1)\Delta x, 0) (2\Delta x) \} ,$$

and equation (2.8) should be identical to the second equation in this note.

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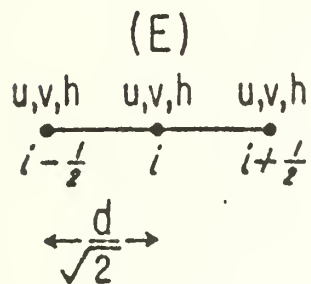
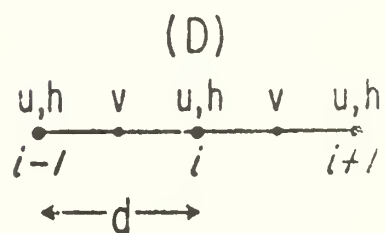
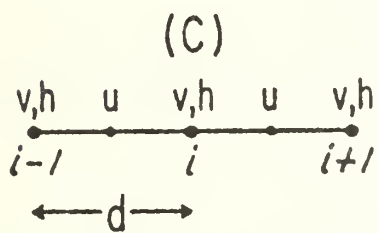
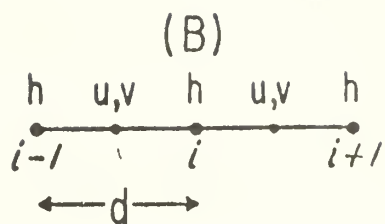
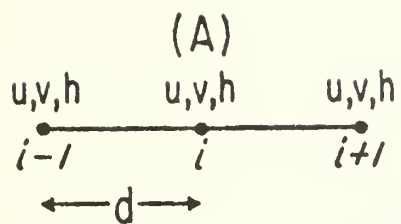


Figure 1

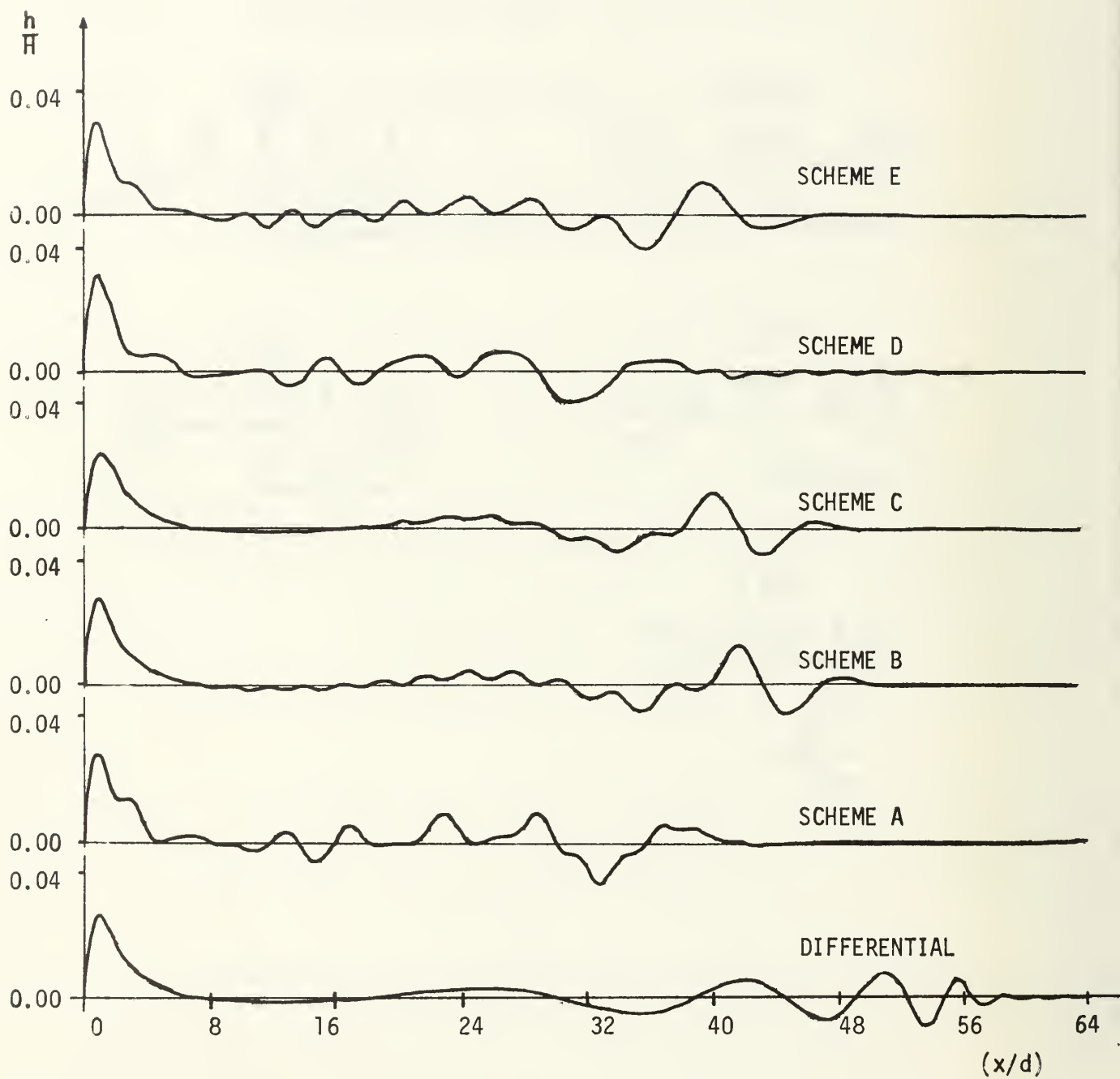


Figure 2

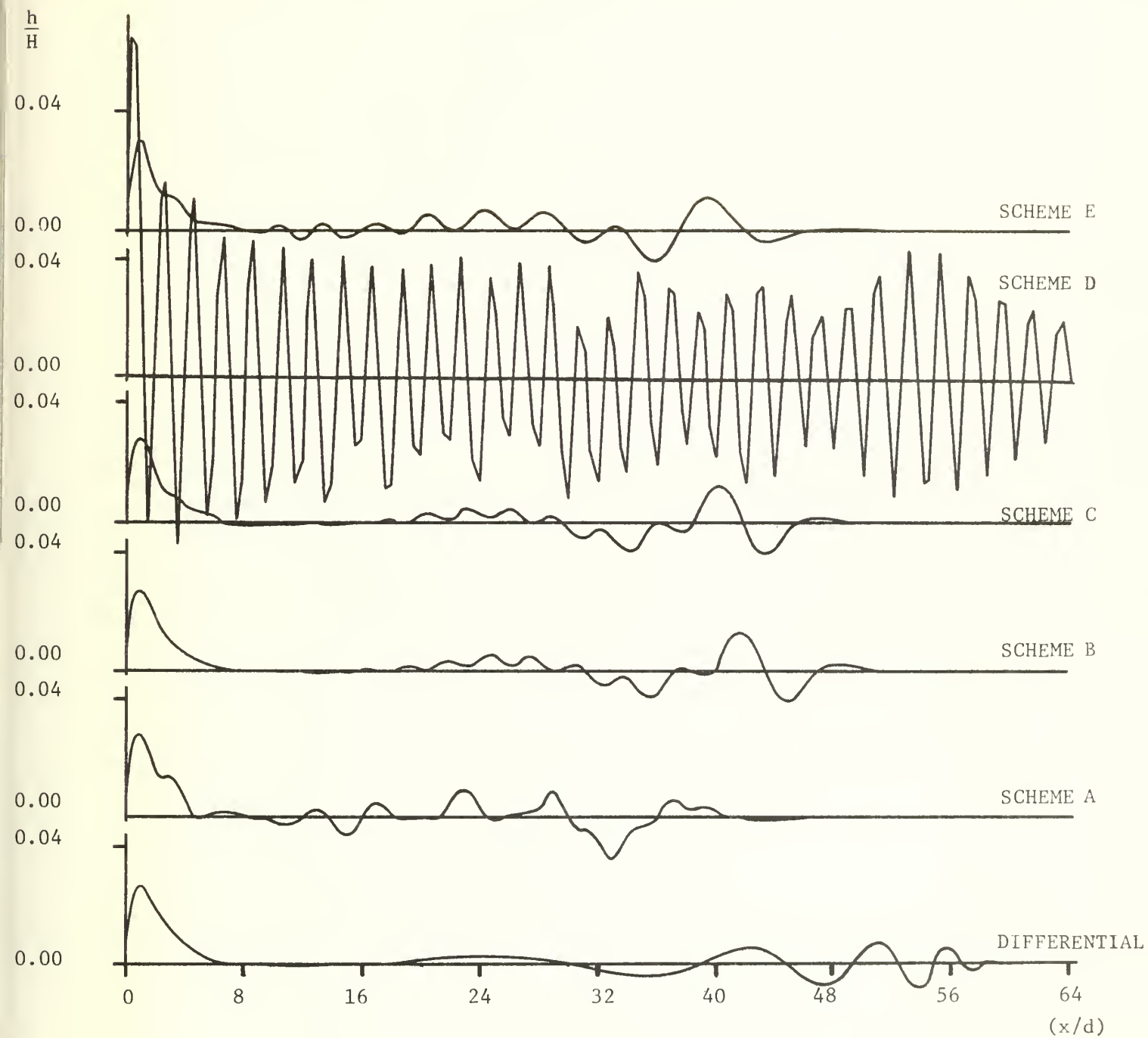


Figure 3

References

- [1] Arakawa, A. and Lamb, V. R., "Computational Design of the Basic Dynamical Processes of the UCLA General Circulation Model", Methods of Computational Physics, Vol. 17, Academic Press, 1977.
- [2] Blumen, W., "Geostrophic Adjustment," Reviews of Geophysics and Space Physics, Vol. 10, No. 2, May 1972, pp 485-528.
- [3] Cahn, A., "An Investigation of the Free Oscillations of a Simple Current System," Journal of Meteorology, Vol. 2, No. 2, 1945, pp 113-119.
- [4] Rossby, C. G., "On the Mutual Adjustment of Pressure and Velocity Distributions in Certain Simple Current Systems," J. Mar. Res., 1, 1937-1938, pp 15-28, 239-263.
- [5] Schoenstadt, A., "The Effect of Spatial Discretization on the Steady-State and Transient Behavior of a Dispersive Wave Equation," J. Comp. Phys., Vol. 23, April 1977, pp 364-379.
- [6] Winninghoff, F., "On the Adjustment Toward a Geostrophic Balance in a Simple Primitive Equation Model with Application to the Problem on Initialization and Objective Analysis," Doctoral Dissertation, UCLA, 1968, 161 pp.

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